Worksheet 5B Solutions

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October 31, 2016

1. (a) What exactly does each of the four operators above measure (more specifically, what do their eigenvalues represent)?

   The $\hat{L}^2$ operator measures the size of the total angular momentum. The $\hat{L_x}$, $\hat{L_y}$, and $\hat{L_z}$ operators measure angular momentum in the x,y,z directions, respectively.

   (b) Given these definitions, do $\hat{L}^2$ and $\hat{L_z}$ commute?

   Using the relations:

$$[\hat{A} + \hat{B}, \hat{C}] = (\hat{A} + \hat{B})\hat{C} - \hat{C}(\hat{A} + \hat{B}) = \hat{A}\hat{C} + \hat{B}\hat{C} - \hat{C}\hat{A} - \hat{C}\hat{B} = [\hat{A}, \hat{C}] + [\hat{B}, \hat{C}] \quad (1)$$

$$[\hat{A}\hat{B}, \hat{C}] = \hat{A}[\hat{B}, \hat{C}] + [\hat{A}, \hat{C}]\hat{B} \quad (2)$$

$$[\hat{L_x}, \hat{L_y}] = i\hbar\hat{L_z} \quad (3)$$

$$[\hat{L_y}, \hat{L_z}] = i\hbar\hat{L_x} \quad (4)$$

$$[\hat{L_z}, \hat{L_x}] = i\hbar\hat{L_y} \quad (5)$$

   we can expand the commutator: $[\hat{L}^2, \hat{L_x}] = [\hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2, \hat{L_x}] = [\hat{L}_x^2, \hat{L_x}] + [\hat{L}_y^2, \hat{L_x}] + [\hat{L}_z^2, \hat{L_x}]$

   Using (2), this becomes: $\hat{L_x}[\hat{L_x}, \hat{L_x}] + [\hat{L_x}, \hat{L_x}]\hat{L_x} + \hat{L_y}[\hat{L_y}, \hat{L_x}] + [\hat{L_y}, \hat{L_x}]\hat{L_y} + \hat{L_z}[\hat{L_z}, \hat{L_x}] + [\hat{L_z}, \hat{L_x}]\hat{L_z}$

   Using (3) and (5) we have: $= -i\hbar\hat{L_y}\hat{L_z} - i\hbar\hat{L_z}\hat{L_y} + i\hbar\hat{L_z}\hat{L_y} + i\hbar\hat{L_y}\hat{L_z} = 0$

   (c) If $[\hat{L}^2, \hat{L_z}] = 0$, Should $\hat{L}^2$ commute with $\hat{L_y}$ and/or $\hat{L_z}$?

   Yes, both $\hat{L_y}$ and $\hat{L_z}$ should also commute with $\hat{L}^2$ based on symmetry considerations. The x direction is arbitrary, so any direction should have the same commutative property.

   (d) If $[\hat{L}^2, \hat{L_x}] = 0$, what does this tell you about the eigenfunctions for these operators and the "measurability" of their corresponding eigenvalues?

   This tells us that they have the same eigenfunctions and thus it is possible to measure both simultaneously with infinite precision.

2. (a) What do the commutator relations between the components of $\hat{L}$ tell us about their eigenfunctions?

   Since they don't commute, this means they have different eigenfunctions.

   (b) How do the commutators between the individual components of angular momentum affect $[\hat{L}^2, \hat{L_x}] = 0$, $[\hat{L}^2, \hat{L_y}] = 0$, and $[\hat{L}^2, \hat{L_z}] = 0$?

   They allow the above commutators to be computed more easily.

(c) Based on the simplicity of the problem, which commutator problem, $[\hat{L^2}, \hat{L_x}] = 0$, $[\hat{L^2}, \hat{L_y}] = 0$, or $[\hat{L^2}, \hat{L_z}] = 0$ looks the easiest? Does switching to spherical coordinates change the resulting commutator relationships discussed above?

The $\hat{L_z}$ operator is simplest so $[\hat{L^2}, \hat{L_z}]$ would be the easiest to compute. Switching to spherical coordinates does not change the resulting commutator relationships.