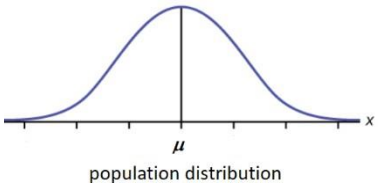
Ch 7 Notes

Ch 7.1 Central Limit Theorem for Sample Means

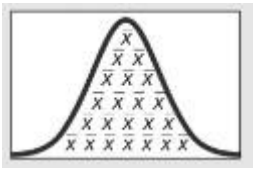
Sample distribution of sample mean:

When sample means ($\bar{x}$) of same size n taken from the same population, the Sample means have the following behavior:

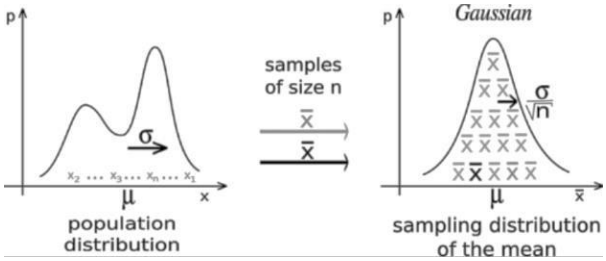
1) If the population distribution of X is normal, the distribution of $\bar{x}$ is always normal for all sample size n.



population distribution


sampling distribution of x-bar

2) When population distribution of X is not normal, The sampling distribution x-bar tends to be a normal distribution. The distribution become closer to normal when sample size increase.



Central Limit Theorem for Sample Mean:

For all sample of the same size n with n > 30, the sampling distribution of $\bar{x}$ can be approximated by a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$.

Note:

-This applies to all distribution of x. If X is normally distributed, n > 30 is not needed. Any n will work.

-The sample should be a Simple Random Sample.

Central limit Theorem

Notation: $\mu_{\bar{x}} = \mu$, $\sigma_{\bar{x}} = \sigma/\sqrt{n}$

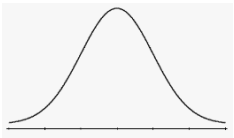
Ex1 A standardized test with scores that are normally distributed with mean μ = 150 and standard deviation σ = 18. A class of 20 students take the test. The mean score $\bar{x}$ of the 20 students are calculated.

a) Is the distribution of mean score ($\bar{x}$) of 20 students Normally distributed?

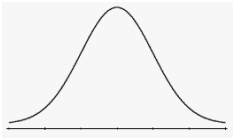
Ans: Yes because the original score is normal.

b) What is the mean and standard deviation of $\bar{x}$?

c) Find the probability that a student's score is greater than 160.



d) Find the probability that the mean score $\bar{x}$ of 20 students is greater than 160.



Ex2: Coke cans are filled so that the actual amounts have a mean of 12 oz and a standard deviation of 0.11 oz. The distribution of amount of coke is unknown.

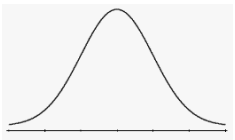
a) Is the distribution of mean amount of coke in 36 cans normally distributed?

Yes, because n > 30, according to CLT, $\bar{x}$ will be normally distributed.

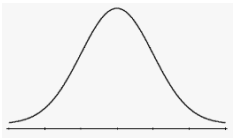
b) What is the mean and standard deviation of $\bar{x}$?

Ans: $\mu_{\bar{x}} = 12$, $\sigma_{\bar{x}} = 0.11/\sqrt{36}$.

c) Find the percent of individual coke with amount between 11.9 to 12.1 oz.



d) Find the percent of mean amount of 36 coke with between 11.9 and 12.1 oz.



Ex3. Annual incomes are known to have a distribution that is skewed to the right. Assume that 20 workers' mean incomes $\bar{x}$ are collected.

a) Will the distribution of mean income $\bar{x}$ be normally distributed?

Ans: No, since X is not normal and n < 30, CLT does not apply, $\bar{x}$ may not be normally distributed.

Ch 7.2 Central Limit Theorem for Sums

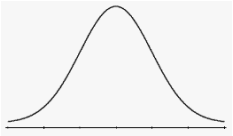
For all sample of the same size n with n > 30, the sampling distribution of $\sum x$ can be approximated by a normal distribution with mean (n)(μ) and standard deviation $\sigma\sqrt{n}$.

Central limit Theorem for Sum $\sum x$
Notation: $\mu_{sum} = (n)(\mu)$, $\sigma_{sum} = \sigma(\sqrt{n})$

-This applies to all distribution of x. If X is normally distributed, n > 30 is not needed. Any n will work.
-The sample should be a Simple Random Sample.

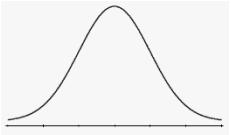
Ex1. An unknown distribution has a mean of 45 and a standard deviation of 8. A sample size of 50- is drawn randomly from the population. .
a) Can Central Limit for Sum be used to model the distribution of the sum of 50 sample? Explain.

b) Find the probability that the sum of the 50 values is more than 2400.



Ex2: An elevator has a maximum weight limit of 5000 lb or 27 passengers. Assume adult males have weights that are normally distributed with a mean of 189 lb. and a standard deviation of 39 lb.
a) Can the sum of 27 passengers' weight be modeled by Normal distribution? If yes, what is the mean and standard deviation of sum of weight of 27 passengers?

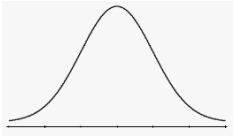
b) Does this elevator appear to be safe? Find the probability that the total weight of 27 passengers exceed the maximum weight limit.



P (sum of 27 > 5000) 185)

Ex3. The passenger load for a water-taxi is 3500 lb. Assume weights of passenger are normally distributed with mean 174 lb. with a standard deviation of 21 lb. Is it safe have a passenger limit of up to 20?
a) Can the sum weight of 20 passengers be modelled by Central limit Theorem of sum? If yes, what is the mean and standard deviation of the weights?

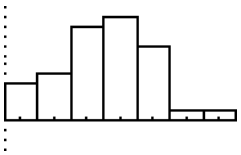
b) Determine the probability that 20 passenger's total weight will exceed the passenger load of 3500 lb. Is the 20-passenger limit safe?



Normality assessment:

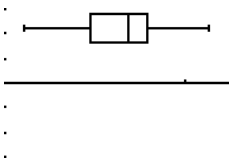
The following procedure can be used to determine if sample data are from a population having a Normal distribution.

1) Histogram: If the histogram departs dramatically from a bell shape, conclude data do not come from a normal distribution.



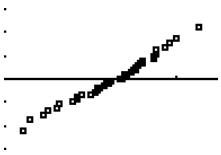
If Histogram from the sample does not departs from a bell shape distribution, conclude the Population may be Normal.

2) Outliers: If there is more than one outlier, conclude data do not come from a normal distribution.



3) Normal Quantile plot: If patterns of the points is reasonably close to a straight line and the points does not show some systematic pattern that is not a straight line pattern, conclude Normal distribution

Normal quantile plot (or Normal probability plot) is a plot of (x, y) where x is the original data and y is the corresponding z-score that is expected from a normal distribution.



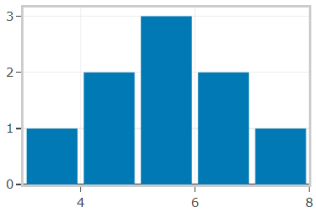
Normal probability plot does not show pattern different from a straight line.

Steps:

1) input data to Sample editor in Statdisk.
2) Data/Normality Assessment/
3) Check that number of outliers are at most 1. Check that Histogram has a approximately bell shape distribution. Check that points on Normal Quantile plot are reasonably close to a straight line.

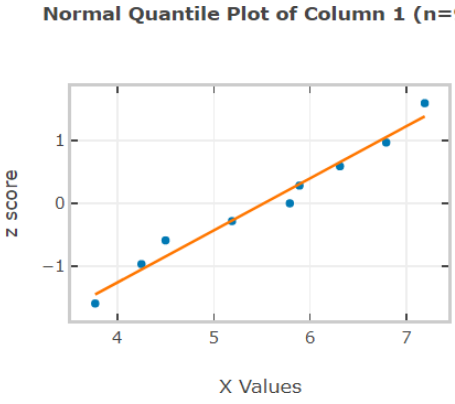
Ex1. Sample question: Do the following values come from a population with normal distribution?
7.19, 6.31, 5.89, 4.5, 3.77, 4.25, 5.19, 5.79, 6.79.

Input data to statdisk.
-number of outliers is 0



Histogram shows relatively symmetrical

-Normal quantile plot shows plot along a linear pattern.



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Final conclusion: the sample are from a population that is Normal.