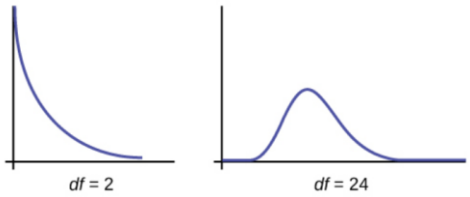
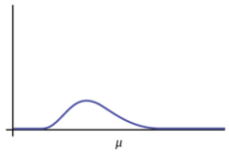
Ch 11 The Chi-square Distribution

Ch 11.1 Facts about Chi-square distribution

Notation for chi-square distribution is χ^2 . It is a distribution with degree of freedom (df = n -1).
Characteristic of chi-square distribution.
i) Shape of the distribution is right skew, non-symmetrical. There is a different chi-square curve for each df. When df > 90, the chi-square curve approximates the normal distribution.



ii) mean μ = df (n-1), σ = $\sqrt{2(df)}$. The mean is located just right of the peak.



iii) χ^2= sum of (n-1) independent, standard normal variable. χ^2 is always positive.
Chi-square distribution calculator:
http://onlinestatbook.com/2/calculators/chi_square_prob.html
The calculator can be used to find area to the right of a chi-square value P(χ^2 > a)
Ex. Find probability that χ^2 is greater than 31 when df = 10.
Enter chi-square = 31, df = 10, calculate.

Ch 11. 3 Test of independence

Contingency table: is a table consisting of frequency counts of categorical data corresponding to two different variables. (One variable is used to categorize rows, the second is used to categorize columns.)
- It is used to calculate conditional probability.
- It is also used to study if row and column variables are independent or independent (associations.)

Test of independence

Approach: Compare expected counts with observed counts in a contingency table to determine association or dependency.

Example:
Sample of 261 college students are surveyed about their favorite snack. Are the choices of snack independent of gender?

	chips	candy	ice-cream	fruit
male	40	14	33	9
female	68	21	56	20

Use the total in each row and column to analysis the expected counts in each cell.

	chips	candy	ice-cream	fruit	total
male	40	14	33	9	96
female	68	21	56	20	165
total	108	35	89	29	261

$E = \frac{(row\ total)(column\ total)}{Grand\ Total}$

Expected count tables.

	Chips	candy	ice-cream	fruit	Total
male	39.7	12.9	32.7	10.7	96.0
female	68.3	22.1	56.3	18.3	165.0
Total	108.0	35.0	89.0	29.0	261

calculate $\chi^2 = \sum \frac{(O-E)^2}{E}$ where O = observed counts, E = expected counts.
Large χ^2 value implies big discrepancy from expected count so conclude row and columns are dependent. There is association between the variables.
Chi-square distribution is used with df = (r-1)(c-1) where r = number of rows, c = number columns.

Requirements:
-Expected counts in each cell is at least 5.
-Sample is simple random sample (SRS).
-The summaries are contingency table of counts.

Null hypothesis(H0) is always no association or independent.

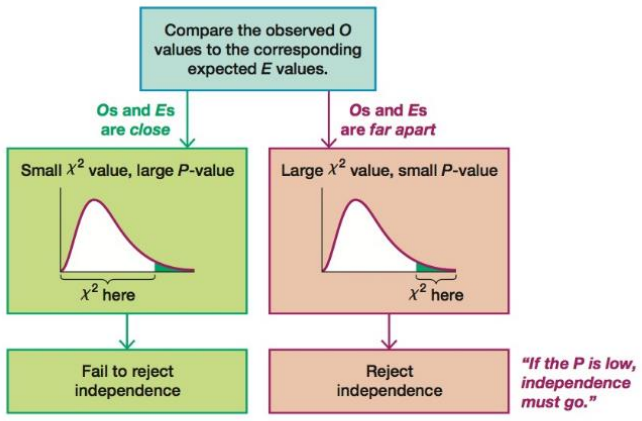


FIGURE 11-4 Relationships Among Key Components in a Test of Independence

A small chi-square value means independence, because the observed counts agree with the expected counts.

The test of independence **is always a right tail test**. because large χ^2 value corresponds to Ha value.
Steps to conduct test of independence:
1) Write H0 and Ha and identify if claim is in H0 or Ha
H0: the row and column variables are independent events. (no associations)
Ha: the row and column variables are dependent events. (has associations)
2) Input the contingency table in columns to Statdisk. Analysis/Contingency table/Enter significance. Select the columns that contain the contingency table. Evaluate. Output: degree of freedom , Test statistics χ^2 and p-value.
3) If p-value ≤ α, Reject H0, conclude dependent. (the row and column variable are associated.)
If p-value > α, fail to reject H0, conclude independent.
4) Conclusion about the claim. If H0 is rejected, there is sufficient evidence, if H0 is failed to be rejected, there is not sufficient evidence.
5) Check that all expected count are at least 5. Pick the cell with the smallest total row count and smallest column count and calculate $E = \frac{(Row\ total)(column\ total)}{Grand\ total}$

Ex1: Results of using nicotine patch and nicotine gum are summarized below. Test the claim results are independent of the method of treatment. Use α = 0.05

	nicotine patch	nicotine gum
Keep smoking	191	263
quit smoking	59	57

1) Write the null hypothesis:
H0: success and failure are independent of treatment.
H1: success and failure are dependent of treatment.
Note: claim is H0.
2) Input the table to Statdisk. Analysis/Contingency Table/, input significance = 0.05, check column 1 , 2 Evaluate. Output: df = 1, Test stat = 2.9, p-value =0.0886.
3) Since 0.0886 > 0.05, fail to reject H0, conclude no association, the result and treatments are independent.
4) There is not sufficient evidence to reject the claim that success and failure are independent of the method of treatment. Conclude they are independent.
5) Check expected count.

	nicotine patch	nicotine gum
Keep smoking	191	263
quit smoking	59	57

Use Mathisfun chi-square calculator

Expected Values:
199.123 254.877
50.8772 65.1228

all expected counts are over 5.

Ex2.
Echinacea experiment was by randomly assign patients to three treatment groups, a placebo group, a 20%-extract group and a 60%-extract group. Counts of infected and not infected for each group is summarized below. Test the claim that infected outcomes are dependent on type of treatments? Use α = 0.05.

	placebo	enchinacea 20%	enchinacea 60%
infected	88	28	29
not infected	15	24	23

1) H0: infected outcome is independent of treatment.
Ha: infected outcome is dependent of treatment.
Note: Claim is Ha.
2) Input the table to Statdisk. Analysis/Contingency, Select column 1, 2, 3, evaluate. Output: df = 2
test stat = χ^2 = 23.19., p-value = 0.
3) Since p-value < 0.05, Reject H0,
4) There is sufficient evidence to support the claim that infected rate is dependent of the type of treatment.
5) Use mathisfun chi-square calculator to find expected counts.

Expected Values:
72.1498 36.4251 36.4251
30.8502 15.5749 15.5749

Requirement for Chi-square test is satisfied.

Test of homogeneity:

When sample data are summarized in a contingency table from different populations, and we can use chi-square test to determine whether those populations have the same proportion of some characteristic being considered, the hypothesis test is known as "test of homogeneity". The method is the same as that of "test of independence."
A chi-square test of homogeneity is a test of the claim that different populations have the same proportions of some characteristics.
Example:
Sample are collected from three populations of workers. Use test of homogeneity to test the claim that choice of transportation are different among the three profession of workers.

	Choice				
Sector	private	public	share	bike	total
Sales	20	1	2	0	23
Technology	10	3	8	2	23
Education	16	2	0	5	23

A test of homogeneity should be used instead of test of independence

The only difference is how samples are collected, the name of the test and how H0 and Ha are written. Everything else are the same as Test of independence.

Ex1. Test the claim that choices of transportation are different among the three profession of workers. Use a significant level of 0.05.

	Choice				
Sector	private	public	share	bike	total
Sales	20	1	2	0	23
Technology	10	3	8	2	23
Education	16	2	0	5	23

1) H0: proportion of the transportation choices are the same among the three professions.

Ha: At least one of the choices are different.

2) Input the table to Statdisk. Analysis/Contingency, Select column 1, 2, 3, 4, evaluate. Output: df = 6 test stat = $\chi^2 = 20.13$., p-value = 0.0026

3) Since p-value < 0.05, Reject H0, conclude different proportions of choices between 3 populations.

4) There is sufficient evidence to support the claim that choices of transportation is different among the three populations of profession.

5) Calculate the expected counts by Mathisfun chi-square calculator.

Expected Values:
15.3333 2 3.33333 2.33333
15.3333 2 3.33333 2.33333
15.3333 2 3.33333 2.33333

Least expected count is $\frac{6\times23}{69}$ =2 < 5, so requirement for chi-square test is not satisfied. The result may not be reliable. More sample should be collected.

Ex2. The Contingency table below summarized a Civil Exam results collected from white candidates and minority candidates. Is there evidence to support the claim that results are different, so the exam is discriminatory? Test the claim that white and minority candidates do not have the same chance of passing the exam. Use a 0.05 significant level.

	Passed	Failed
White candidates	15	14
Minority candidates	7	26

1) H0: White and minority candidates have same chance of passing the exam

Ha: White and minority candidate do not have the same chance of passing the exam

Note: claim is Ha

2) Input the table to Statdisk. Analysis/Contingency Table. Enter significance, select column 1, 2 evaluate. Output: df = 1 test stat = $\chi^2 = 6.28$, p-value = 0.0122,

3) Since 0.0122 < 0.05, Reject H0. Conclude the two population has different chance of passing.

4) There is sufficient evidence to support the claim that white and minority candidates do not have the same chance of passing the exam.

5) Check requirement by calculating expected counts using Mathisfun chi-square calculator.

Expected Values:
10.2903 18.7097
11.7097 21.2903

All expected counts are at least 5, hence chi-square test requirement is satisfied.